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M10 Honours: Section 2.1 Quadratic Functions $y = ax^2 + bx + c$

1. Indicate the values of "a" "b" and "c" in each of the following equations:

a) $y = x^2 - 2x - 5$ $y = ax^2 + bx + c$ $a = 1 \quad b = -2 \quad c = -5$	b) $y = \frac{1}{2}x^2 + 5$ $a = \frac{1}{2} \quad b = 0 \quad c = 5$	c) $y - x^2 + 2 = 0$ $y = x^2 - 2$ $a = 1 \quad b = 0 \quad c = -2$
d) $y = x(x-7)$ $y = x^2 - 7x$ $a = 1 \quad b = -7 \quad c = 0$	e) $f(x) = x^2 + 1$ $f(x) = y$ $y = x^2 + 1$ $a = 1 \quad b = 0 \quad c = 1$	f) $y = -3(x + \frac{4}{3})^2 - 10$ $y = -3(x^2 + \frac{8}{3}x + \frac{16}{9}) - 10$ $y = -3x^2 - 8x - \frac{16}{3} - 10$ $y = -3x^2 - 8x - \frac{46}{3}$ $a = -3 \quad b = -8 \quad c = -\frac{46}{3}$

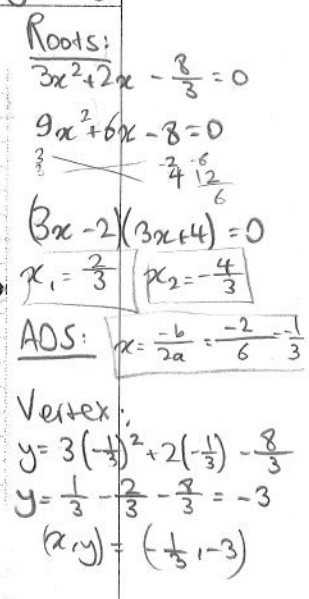
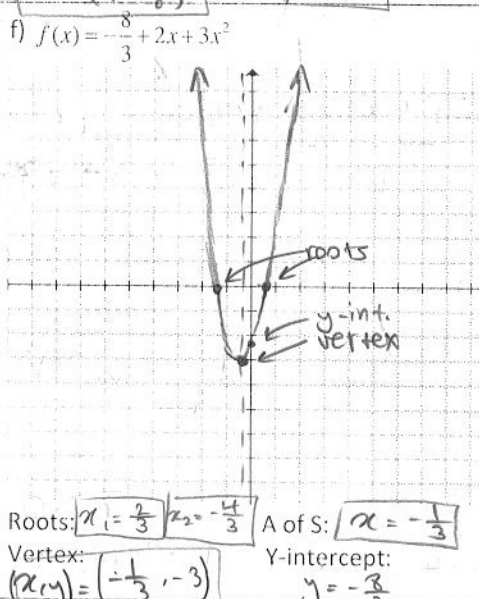
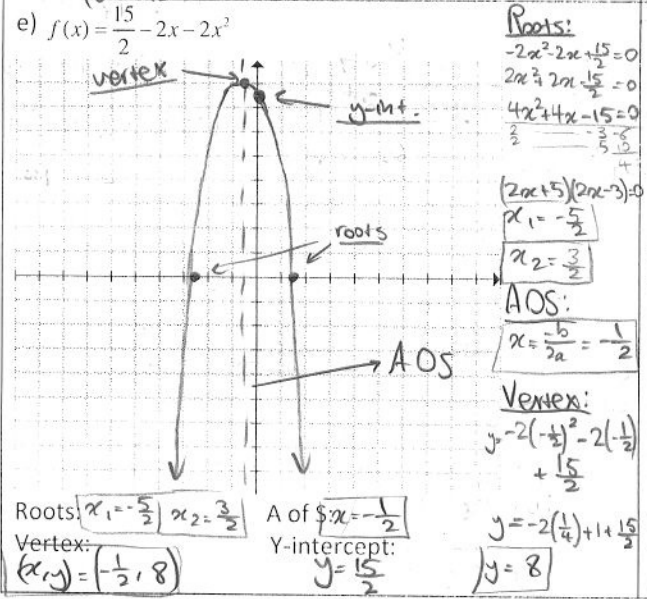
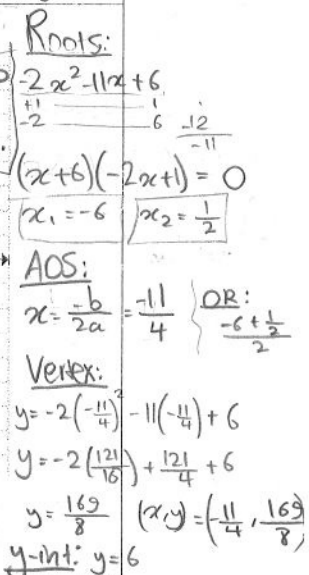
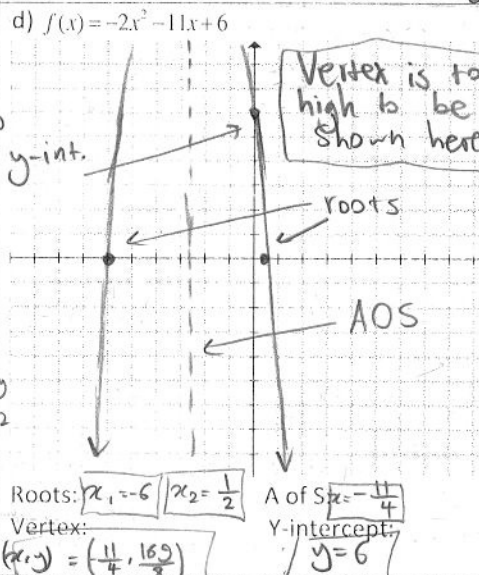
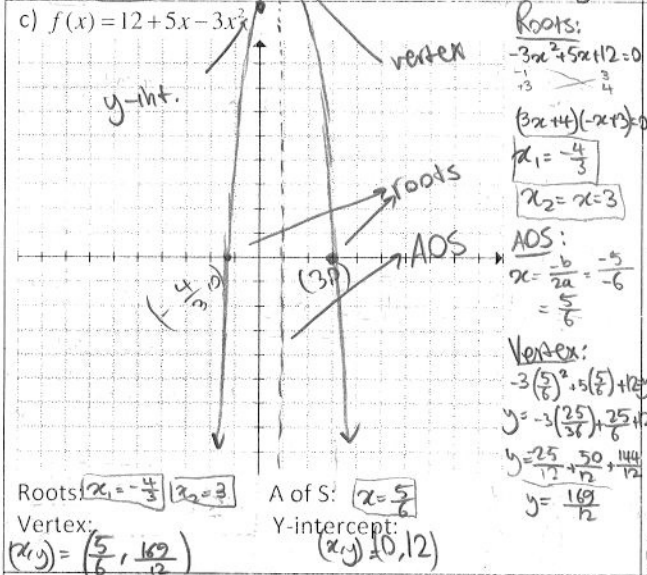
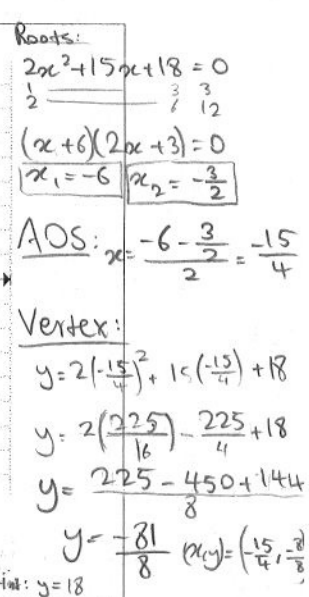
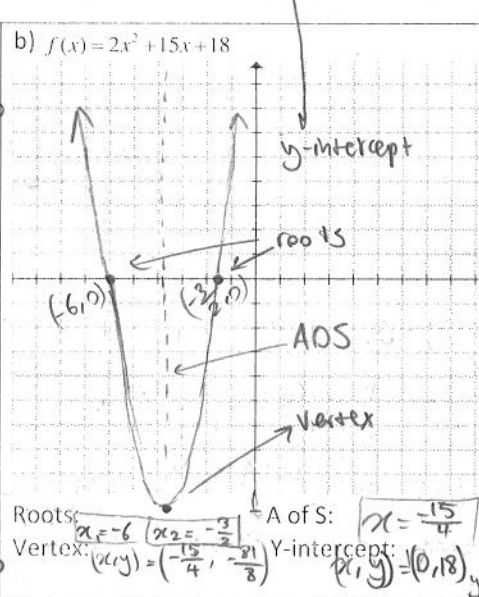
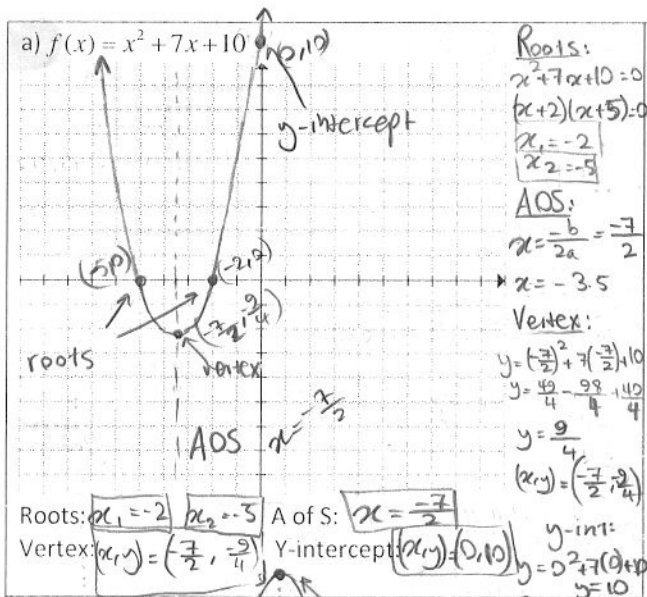
2. Solve each of the following quadratic equations. Provide your answers in exact form

a) $5x - 1 = 2x^2$ $2x^2 - 5x + 1 = 0$ First approach: (quadratic formula) $2x^2 - 5x + 1 = 0$ $ax^2 + bx + c = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{5 \pm \sqrt{25 - 4(2)}}{2(2)}$ $x = \frac{5 \pm \sqrt{25 - 8}}{4}$ $x_1 = \frac{5 + \sqrt{17}}{4} \quad x_2 = \frac{5 - \sqrt{17}}{4}$	Second approach: (completing the square) $2x^2 - 5x + 1 = 0$ $2(x^2 - \frac{5}{2}x) + 1 = 0$ $2(x^2 - \frac{5}{2}x + \frac{25}{16}) + 1 = \frac{25}{8}$ $2(x - \frac{5}{4})^2 = \frac{17}{8}$ $x - \frac{5}{4} = \pm \sqrt{\frac{17}{16}}$ $x_1 = \frac{5 + \sqrt{17}}{4} \quad x_2 = \frac{5 - \sqrt{17}}{4}$	b) $8x + 8 = 12x^2$ $12x^2 - 8x - 8 = 0$ $4(3x^2 - 2x - 2) = 0$ $3x^2 - 2x - 2 = 0$ $ax^2 + bx + c = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{2 \pm \sqrt{4 - 4(3(-2))}}{2(3)}$ $x = \frac{2 \pm \sqrt{4 + 24}}{6}$ $x = \frac{2 \pm \sqrt{28}}{6}$ $x = \frac{2 \pm 2\sqrt{7}}{6}$ $x_1 = \frac{1 + \sqrt{7}}{3} \quad x_2 = \frac{1 - \sqrt{7}}{3}$
c) $x^2 - 5x + 3 = 0$ Quadratic Formula $ax^2 + bx + c = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{5 \pm \sqrt{25 - 12}}{2}$ $x = \frac{5 \pm \sqrt{13}}{2}$ $x_1 = \frac{5 + \sqrt{13}}{2} \quad x_2 = \frac{5 - \sqrt{13}}{2}$	Completing the square $x^2 - 5x + 3 = 0$ $x^2 - 5x + \frac{25}{4} + 3 = \frac{25}{4}$ $(x - \frac{5}{2})^2 = \frac{25}{4} - \frac{12}{4}$ $x - \frac{5}{2} = \pm \sqrt{\frac{13}{4}}$ $x = \frac{5}{2} \pm \frac{\sqrt{13}}{2}$ $x_1 = \frac{5 + \sqrt{13}}{2} \quad x_2 = \frac{5 - \sqrt{13}}{2}$	d) $6x + 6 = 15x^2$ $15x^2 - 6x - 6 = 0$ $ax^2 + bx + c = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{6 \pm \sqrt{36 - 4(15(-6))}}{30}$ $x = \frac{6 \pm \sqrt{36 + 900}}{30}$ $x = \frac{6 \pm \sqrt{936}}{30}$ $x = \frac{6 \pm 6\sqrt{11}}{30}$ $x_1 = \frac{1 + \sqrt{11}}{5} \quad x_2 = \frac{1 - \sqrt{11}}{5}$

3. Factor each of the following quadratic functions and find i) the Coordinates of the Roots, ii) the Equation of the Axis of Symmetry, iii) Coordinates of the Vertex, iii) Domain and Range

a) $y = x^2 + 3x - 18$ $D: x \in \mathbb{R}$ <u>Roots:</u> $x^2 + 3x - 18 = 0$ $(x-3)(x+6) = 0$ $x_1 = 3$ $x_2 = -6$ <u>A of S:</u> $x = \frac{-b}{2a} = \frac{-3}{2}$ <u>Coordinates of vertex:</u> $y = (-\frac{3}{2})^2 + 3(-\frac{3}{2}) - 18$ $y = \frac{9}{4} - \frac{18}{2} - \frac{72}{4}$ $y = -\frac{81}{4}$ $(x, y) = (-\frac{3}{2}, -\frac{81}{4})$ <u>Roots:</u> $x_1 = 3$ $x_2 = -6$ <u>A of S:</u> $x = -\frac{3}{2}$ <u>Vertex:</u> $(x, y) = (-\frac{3}{2}, -\frac{81}{4})$ <u>Range:</u> $y \geq -\frac{81}{4}$	b) $y = 2x^2 + 3x - 2$ <u>Roots:</u> $2x^2 + 3x - 2 = 0$ $(2x-1)(x+2) = 0$ $x_1 = \frac{1}{2}$ $x_2 = -2$ <u>A of S:</u> $x = \frac{-b}{2a} = \frac{-3}{4}$ <u>Vertex:</u> $y = 2(-\frac{3}{4})^2 + 3(-\frac{3}{4}) - 2$ $y = \frac{9}{8} - \frac{9}{4} - 2$ $y = \frac{9}{8} - \frac{18}{8} - \frac{16}{8}$ $y = -\frac{25}{8}$ $(x, y) = (-\frac{3}{4}, -\frac{25}{8})$ <u>Roots:</u> $x_1 = \frac{1}{2}$ $x_2 = -2$ <u>A of S:</u> $x = -\frac{3}{4}$ <u>Vertex:</u> $(x, y) = (-\frac{3}{4}, -\frac{25}{8})$ <u>Range:</u> $y \geq -\frac{25}{8}$	c) $y = -x^2 - 12x - 35$ <u>Roots:</u> $-x^2 - 12x - 35 = 0$ $x^2 + 12x + 35 = 0$ $(x+5)(x+7) = 0$ $x_1 = -5$ $x_2 = -7$ <u>A of S:</u> $x = \frac{-b}{2a} = \frac{-12}{-2} = 6$ <u>Vertex:</u> $y = -6^2 - 12(-6) - 35$ $y = -36 + 72 - 35$ $y = 1$ $(x, y) = (-6, 1)$ <u>Roots:</u> $x_1 = -5$ $x_2 = -7$ <u>A of S:</u> $x = -6$ <u>Vertex:</u> $(x, y) = (-6, 1)$ <u>Range:</u> $y \leq 1$
d) $y = x^2 + \frac{5}{2}x - \frac{3}{2}$ <u>Vertex:</u> <u>Roots:</u> $x^2 + \frac{5}{2}x - \frac{3}{2} = 0$ $2x^2 + 5x - 3 = 0$ $(2x-1)(x+3) = 0$ $x_1 = -3$ $x_2 = \frac{1}{2}$ <u>A of S:</u> $x = \frac{-b}{2a} = \frac{-\frac{5}{2}}{2} = -\frac{5}{4}$ <u>Vertex:</u> $y = (-\frac{5}{4})^2 + \frac{5}{2}(-\frac{5}{4}) - \frac{3}{2}$ $y = \frac{25}{16} - \frac{25}{8} - \frac{3}{2}$ $y = \frac{25}{16} - \frac{50}{16} - \frac{24}{16}$ $y = -\frac{49}{16}$ $(x, y) = (-\frac{5}{4}, -\frac{49}{16})$ <u>Roots:</u> $x_1 = -3$ $x_2 = \frac{1}{2}$ <u>A of S:</u> $x = -\frac{5}{4}$ <u>Vertex:</u> $(x, y) = (-\frac{5}{4}, -\frac{49}{16})$ <u>Range:</u> $y \geq -\frac{49}{16}$	e) $y = 6x^2 + 13x - 5$ <u>Vertex:</u> <u>Roots:</u> $6x^2 + 13x - 5 = 0$ $(2x+5)(3x-1) = 0$ $x_1 = -\frac{5}{2}$ $x_2 = \frac{1}{3}$ <u>A of S:</u> $x = \frac{-b}{2a} = \frac{-13}{12}$ <u>Vertex:</u> $y = 6(-\frac{13}{12})^2 + 13(-\frac{13}{12}) - 5$ $y = 6(\frac{169}{144}) - \frac{169}{12} - 5$ $y = \frac{169}{24} - \frac{338}{24} - \frac{120}{24}$ $y = -\frac{289}{24}$ $(x, y) = (-\frac{13}{12}, -\frac{289}{24})$ <u>Roots:</u> $x_1 = -\frac{5}{2}$ $x_2 = \frac{1}{3}$ <u>A of S:</u> $x = -\frac{13}{12}$ <u>Vertex:</u> $(x, y) = (-\frac{13}{12}, -\frac{289}{24})$ <u>Range:</u> $y \geq -\frac{289}{24}$	f) $y = 15x^2 - 7x - 2$ <u>Vertex:</u> <u>Roots:</u> $15x^2 - 7x - 2 = 0$ $(3x-2)(5x+1) = 0$ $x_1 = \frac{2}{3}$ $x_2 = -\frac{1}{5}$ <u>A of S:</u> $x = \frac{-b}{2a} = \frac{7}{30}$ <u>Vertex:</u> $y = 15(\frac{7}{30})^2 - 7(\frac{7}{30}) - 2$ $y = 15(\frac{49}{900}) - \frac{49}{30} - 2$ $y = \frac{49}{60} - \frac{49}{30} - \frac{120}{60}$ $y = \frac{49 - 98 - 120}{60}$ $y = -\frac{169}{60}$ $(x, y) = (\frac{7}{30}, -\frac{169}{60})$ <u>Roots:</u> $x_1 = \frac{2}{3}$ $x_2 = -\frac{1}{5}$ <u>A of S:</u> $x = \frac{7}{30}$ <u>Vertex:</u> $(x, y) = (\frac{7}{30}, -\frac{169}{60})$ <u>Range:</u> $y \geq -\frac{169}{60}$
g) $y = 32x^2 - 60x - 27$ <u>Vertex:</u> <u>Roots:</u> $32x^2 - 60x - 27 = 0$ $(4x-9)(8x+3) = 0$ $x_1 = \frac{9}{4}$ $x_2 = -\frac{3}{8}$ <u>A of S:</u> $x = \frac{-b}{2a} = \frac{60}{64} = \frac{15}{16}$ <u>Vertex:</u> $y = 32(\frac{15}{16})^2 - 60(\frac{15}{16}) - 27$ $y = 32(\frac{225}{64}) - \frac{900}{8} - 27$ $y = \frac{225}{2} - \frac{225}{2} - 27$ $y = -27$ $(x, y) = (\frac{15}{16}, -27)$ <u>Roots:</u> $x_1 = \frac{9}{4}$ $x_2 = -\frac{3}{8}$ <u>A of S:</u> $x = \frac{15}{16}$ <u>Vertex:</u> $(x, y) = (\frac{15}{16}, -27)$ <u>Range:</u> $y \geq -27$	h) $y = \frac{1}{2}x^2 + \frac{1}{2}x - 6$ <u>Vertex:</u> <u>Roots:</u> $\frac{1}{2}x^2 + \frac{1}{2}x - 6 = 0$ $x^2 + x - 12 = 0$ $(x-3)(x+4) = 0$ $x_1 = 3$ $x_2 = -4$ <u>A of S:</u> $x = \frac{-b}{2a} = \frac{-1}{2} = -\frac{1}{2}$ <u>Vertex:</u> $y = \frac{1}{2}(-\frac{1}{2})^2 + \frac{1}{2}(-\frac{1}{2}) - 6$ $y = \frac{1}{8} - \frac{1}{4} - 6$ $y = \frac{1-2-48}{8}$ $y = -\frac{49}{8}$ $(x, y) = (-\frac{1}{2}, -\frac{49}{8})$ <u>Roots:</u> $x_1 = 3$ $x_2 = -4$ <u>A of S:</u> $x = -\frac{1}{2}$ <u>Vertex:</u> $(x, y) = (-\frac{1}{2}, -\frac{49}{8})$ <u>Range:</u> $y \geq -\frac{49}{8}$	i) $y = x^2 + \frac{1}{6}x - \frac{1}{6}$ <u>Vertex:</u> <u>Roots:</u> $x^2 + \frac{1}{6}x - \frac{1}{6} = 0$ $6x^2 + x - 1 = 0$ $(2x+1)(3x-1) = 0$ $x_1 = -\frac{1}{2}$ $x_2 = \frac{1}{3}$ <u>A of S:</u> $x = \frac{-b}{2a} = \frac{-\frac{1}{6}}{2} = -\frac{1}{12}$ <u>Vertex:</u> $y = (-\frac{1}{12})^2 + \frac{1}{6}(-\frac{1}{12}) - \frac{1}{6}$ $y = \frac{1}{144} - \frac{1}{72} - \frac{1}{6}$ $y = \frac{1-2-24}{144}$ $y = -\frac{25}{144}$ $(x, y) = (-\frac{1}{12}, -\frac{25}{144})$ <u>Roots:</u> $x_1 = -\frac{1}{2}$ $x_2 = \frac{1}{3}$ <u>A of S:</u> $x = -\frac{1}{12}$ <u>Vertex:</u> $(x, y) = (-\frac{1}{12}, -\frac{25}{144})$ <u>Range:</u> $y \geq -\frac{25}{144}$

4. Graph the quadratic functions and label on the graph the Roots, AOS, Vertex, and Y-intercepts:



gmk

5. Determine the coordinates of the vertex and eqn of the AOS for the parabola $y = 3(x-20)(x+22)$

$$y = 3(x-20)(x+22)$$

Roots: $x_1 = 20$
 $x_2 = -22$

Axis of Symmetry:
 $x = \frac{20-22}{2} = -1$
 $x = -1$

Vertex:
 $y = 3(-1-20)(-1+22) = -441 \times 3$
 $(x,y) = (-1, -1323)$

6. A pebble is dropped from a bridge into a river at height "h" meters above. Let "t" be the number of seconds after the release. If $h(t) = 65 - 4.9t^2$, then

- a) How high is the pebble after 3 seconds?

$$h(3) = 65 - 4.9(3)^2 = 65 - 44.1 = 20.9 \text{ m.}$$

- b) When is the pebble at the maximum height?

Vertex: AOS: 0
 $y = 65 - 0 = 65$
0 seconds at max height $(t,h) = (0,65)$

- c) What is the maximum height?

65 meters AOS: 0 Vertex: (0,65)

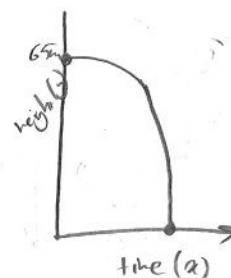
- d) When will the pebble hit the ground?

Ground height: 0 meters $0 = 65 - 4.9t^2$
 $t = \frac{\pm \sqrt{4(4.9 \times 65)}}{-9.8} = \frac{\pm 127.4}{-9.8}$

- e) What is the domain and range of this scenario? time cannot be negative, so

D: $0 \leq t \leq 130$
R: $0 \leq h \leq 65$

$t_1 = \frac{127.4}{-9.8}$ (rej.)
 $t_2 = \frac{127.4}{9.8} = 130$



Always check
Max/Min for
Range.

7. Tom throws a football from the top of his building. The height of the ball is given by the formula: $h(t) = -3t^2 + 60t + 132$, where "h" is the height of the football and "t" is the number of seconds after the throw.

- a) How high was the building?

height of building: $h(0) = -3(0) + 60(0) + 132 = 132 \text{ meters}$

- b) What is the maximum height of the ball?

432 meters (occurs on the 10th second)

- c) For how many seconds will the ball be above the height of 82m?

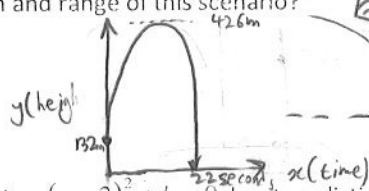
$$-3t^2 + 60t + 132 > 82 \quad -3t^2 + 60t + 50 > 0$$

- d) When will the ball be falling to 50m?

$$50 = -3t^2 + 60t + 132 \quad 82$$

- e) What is the domain and range of this scenario?

D: $0 \leq t \leq 22$
R: $0 \leq h \leq 426$



b/c at 0 seconds, the ball is on top of the building.

$0 \leq t < \frac{30 + 5\sqrt{42}}{3} \approx 20.8 \text{ seconds}$

Max: $-3(t^2 - 20t - 42) = h(t)$
 $-3((t-10)^2 - 142) = h(t)$
 $-3(t-10)^2 + 426 = h(t)$
MAX: 426

8. If the quadratic equation $(x-2)^2 + k = 0$ has two distinct real roots, then what is the range of "k"?

(Multiple choice, circle one) Justify your answer.

a) $k > 2$

b) $k < 0$

c) $k \leq 0$

d) $k \leq 4$

$$(x-2)^2 + k = 0$$

$$x^2 - 4x + 4 + k = 0$$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a=1 \quad b=-4 \quad c=(4+k)$

For $(x-2)^2 + k$ to have two functions, the discriminant needs to be positive. If it is negative, there will be no roots, if it is 0, there will be only 1 root.

$$b^2 - 4ac > 0$$

$$16 - 4(4+k) > 0$$

$$-4k - 16 + 16 > 0$$

$k < 0$

9. Find the values of "A" and "B" if $x^2 + 6x + 16 = (x + A)^2 + B$

match terms $\Rightarrow$
(match because A & B have to work for any value of x)

$$x^2 + 6x + 16 = x^2 + 2Ax + A^2 + B$$

$$\boxed{A=3}$$

$$\boxed{B=7}$$

$$6x = 2Ax$$

$$\boxed{A=3}$$

$$A^2 + B = 16$$

$$3^2 + B = 16$$

$$\boxed{B=7}$$

10. Find the values of "A" and "B" if $x^2 - 10x + 27 = (x + A)^2 + B$

$$x^2 - 10x + 27 = x^2 + 2Ax + A^2 + B$$

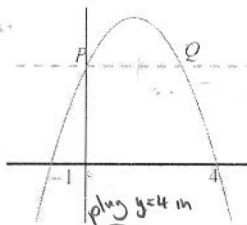
$$-10x = 2Ax$$

$$\boxed{A=-5}$$

$$(-5)^2 + B = 27$$

$$\boxed{B=2}$$

11. The figure below shows the graph of $y = -x^2 + px + q$. The graph cuts the y-axis at point "P". A horizontal line is drawn through points "P" and "Q". What are the coordinates of point "Q"?



$$y = -x^2 + px + q$$

$$\text{Roots: } (x, y) = (-1, 0), (4, 0)$$

$$\text{point "Q" = } (3, 4)$$

$$0 = -(4)^2 + p(4) + q$$

$$0 = -16 + 4p + q$$

$$4p + q = 16$$

$$4p + (1+p) = 16$$

$$\boxed{p=3}$$

$$\boxed{q=4}$$

$$y = -x^2 + 3x + 4$$

$$y = -(0)^2 + 3(0) + 4$$

$$P = (0, 4)$$

$$-x^2 + 3x = 0$$

$$x(-x+3) = 0$$

$$\boxed{Q = (3, 4)}$$

$$0 = -(-1)^2 + p(-1) + q$$

$$0 = -1 - p + q$$

$$-p + q = 1 \quad q = 1 + p$$

12. If the quadratic function $y = ax^2 + bx + c$ has two equal roots and opens up, then which of the following statements are correct?

$$\text{i) } a > 0$$

positive "a" indicates that graph will open up!

$$\text{ii) } c > 0$$

If "c" were greater than 0, there would be no roots and the graph would be shifted up.

$$\text{iii) } b^2 - 4ac > 0$$

If so, the roots would not be identical. If $b^2 - 4ac = 0$ then there will only be one root or "two identical roots".

13. If $y = (x-2)^2$ and $y = 2x+1$ intersect at points (x_1, y_1) and (x_2, y_2) , then which of the following quadratic functions has the roots at x_1 and x_2 ?

$$\text{a) } y = x^2 - 6x + 3$$

$$\text{b) } y = x^2 - 2x + 3$$

$$\text{c) } y = x^2 - 6x + 1$$

$$\text{d) } y = x^2 - 2x + 1$$

$$y = x^2 - 4x + 4$$

$$y = 2x + 1$$

$$x^2 - 4x + 4 = 2x + 1$$

$$x^2 - 6x + 3 = 0$$

$$x = \frac{6 \pm \sqrt{36-12}}{2}$$

$$x = \frac{6 \pm 2\sqrt{6}}{2} = 3 \pm \sqrt{6}$$

$$x_1 = 3 + \sqrt{6} \quad x_2 = 3 - \sqrt{6}$$

$$y_1 = 2(3 + \sqrt{6}) + 1 = 7 + \sqrt{6}$$

$$y_2 = 2(3 - \sqrt{6}) + 1 = 7 - \sqrt{6}$$

$$(x_1, y_1) = (3 + \sqrt{6}, 7 + \sqrt{6})$$

$$(x_2, y_2) = (3 - \sqrt{6}, 7 - \sqrt{6})$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = 3 \pm \sqrt{6}$$

$$-\frac{b}{2a} = 3 \quad \frac{\sqrt{b^2 - 4ac}}{2a} = \sqrt{6}$$

$$-\frac{6}{2} = 3 \quad \frac{\sqrt{36-12}}{2} = \sqrt{6}$$

$$y = x^2 - 6x + 3$$

14. Determine all values of "k" with $k \neq 0$ for which the parabola has its vertex on the x-axis.

$$y = kx^2 + (5k+3)x + (6k+5)$$

$$\begin{aligned} a &= k \\ b &= 5k+3 \\ c &= 6k+5 \end{aligned}$$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\sqrt{b^2 - 4ac}$ has to be 0 in order for the quadratic function to have only 1 root.

$$(5k+3)^2 - 4(k)(6k+5) = 0$$

$$25k^2 + 30k + 9 - 4(6k^2 + 5k) = 0$$

$$k^2 + 10k + 9 = 0 \quad (k+9)(k+1) = 0$$

$$k_1 = -1 \quad k_2 = -9$$

15. Point "A" is the vertex of the parabola $y = x^2 + 2$, point "B" is the vertex of the parabola

$$y = x^2 - 6x + 7, \text{ and "O" is the origin. Determine the area of } \triangle AOB.$$

Vertex of $y = x^2 + 2$
MIN point: 2

(A) vertex: (0, 2)

Area (shoelace method)

$$\text{Area} = \frac{m-n}{2}$$

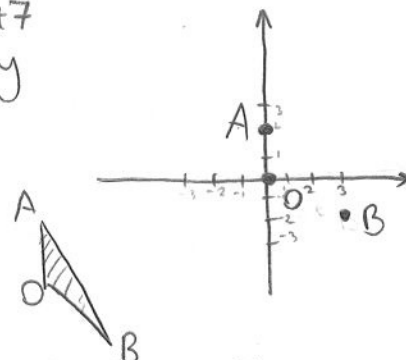
$$\begin{array}{c|c|c} 0 & 2 & 0 \\ 3 & -2 & 0 \\ 0 & 0 & 2 \end{array}$$

$$A = \frac{6}{2} = 3 \text{ units}^2$$

Vertex of $y = x^2 - 6x + 7$
set min: $(x-3)^2 - 2 = y$

MIN: -2

(B) vertex: (3, -2)



16. Consider the function $f(x) = 2x^2 - 4x + c$. What value of "c" maximizes the product of the roots of the function, given that at least one root is real?

Product of roots:

$$C = 2$$

$$\left(\frac{2 + \sqrt{16-8c}}{2} \right) \left(\frac{2 - \sqrt{16-8c}}{2} \right) = \frac{4 - 2(2-c)}{4} = \frac{c}{2}$$

So the max. value of C is 2

$$2x^2 - 4x + c = 0$$

$$x = \frac{4 \pm \sqrt{16-8c}}{4} = \frac{2 \pm \sqrt{2(2-c)}}{2}$$

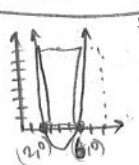
if $c > 2$, both roots will become imaginary & unreal.

17. Challenge: square OPQR has vertices O(0,0), P(0,8), Q(8,8) and R(8,0). The parabola with equation

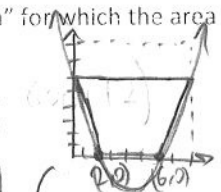
$y = a(x-2)(x-6)$ intersects the sides of the square OPQR at points "K", "L", "M", and "N". Determine all the values of "a" for which the area of the trapezoid KLMN is 36.

first possibility

second possibility:



$$A_{\text{trapezoid}} = \left(\frac{5+4}{2} \right) (8) = 36$$



y-intercept would need to be (0, 6):

$$y = a(x^2 - 8x + 12)$$

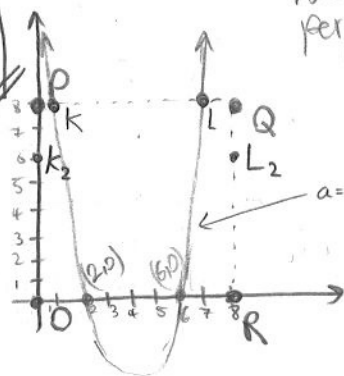
plug in (x,y) (0,6)

$$6 = a(12)$$

$$a = \frac{1}{2}$$

$$A_{\text{trapezoid}} = \left(\frac{8+4}{2} \right) (6) = 36$$

$$\begin{aligned} a_1 &= \frac{1}{2} \\ a_2 &= \frac{32}{9} \end{aligned}$$



NOTE: parabola can't possibly open down. MAX Area $A < 32$

In this case, the graph would need to pass through points $(x,y) = (\frac{3}{2}, 8)$.

Plug in $(x,y) = (\frac{3}{2}, 8)$ into the quadratic to get the value of a for which the trapezoid will have area of 36:

$$y = a(x^2 - 8x + 12) \quad 8 = a\left(\frac{9}{4} - 12 + 12\right) \quad a = \frac{8}{\frac{9}{4} - 12 + 12} = \frac{32}{9}$$